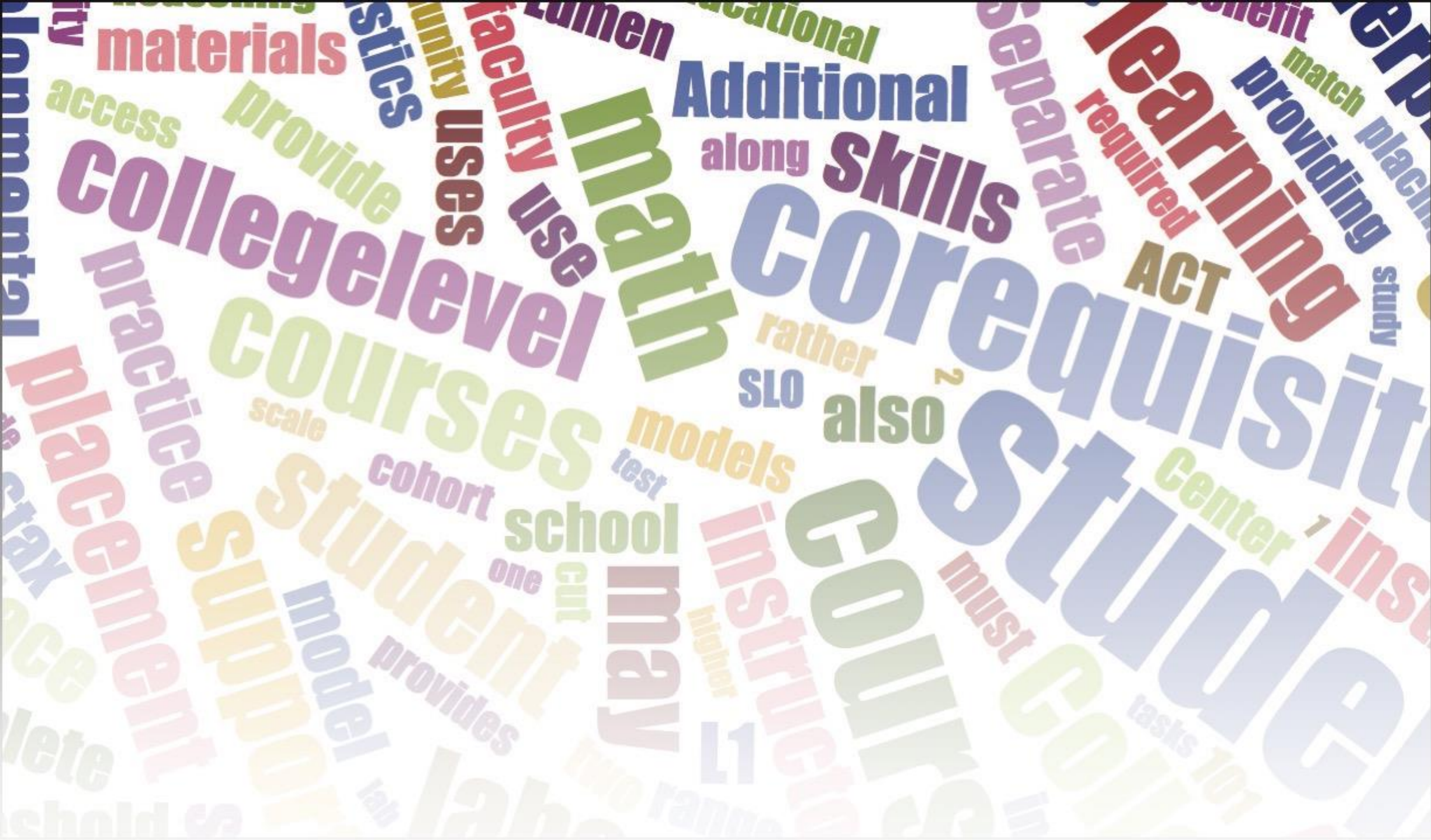

Implementation Strategies for Corequisite Offerings in Mathematics

Missouri Department of Higher Education Corequisite at Scale Taskforce

Spring 2018

Table of Contents

What Is a Corequisite?

The corequisite model places underprepared students into college-level courses with additional support. The corequisite model should not be viewed as the practice of enrolling students in a pre-requisite course and a general education course concurrently. Corequisites should avoid a focus on duplicating high school content. Rather, corequisites emphasize providing just-in-time support of college-level content and expectations. Additionally, the corequisite model is not recommended for dual-credit or online courses.

Traditional two or three semester developmental sequences were designed to provide underprepared students more time to master prerequisite content. However, these longer sequences introduced multiple stop-out points for students and increased the time before the content was put to use in the college-level course. Also, not all of the content learned in the prerequisite courses directly applies to the math pathway college-level course. Corequisite models have increased student completion rates in gateway mathematics courses by providing support to underprepared students precisely when and where it is needed.

Missouri House Bill 1042 was signed into law in 2012 requiring Missouri's higher education intuitions to replicate best practices in remediation. Corequisite instruction was introduced for this reason.

Underprepared Students

Underprepared students can be described as those who have "potential for college success when appropriate educational enrichment and support services are provided" (Arendale, 2007, p.13). Support services, as defined by the National Association for Developmental Education, may include a "comprehensive process that focuses on the intellectual, social and emotional growth and development" of underprepared students.

Corequisite Models

Blended Structure

The blended structure creates a learning environment where underprepared students are enrolled in the same general education course as those students identified as academically prepared. Underprepared students have the same learning experience as the prepared students but also receive additional just-in-time support through a mandatory corequisite course. Separate courses provide structured support before or after the college-level course and can be scheduled on the same day or on opposite days. Credit may be assigned to both the college-level course and the corequisite course. Students may receive separate grades for each course. Corequisite courses may be taught by two different instructors.

Cohort Structure

The cohort structure provides designated sections of the general education course specifically for underprepared students. The additional just-in-time support may be through a separate course or may be embedded in the general education course. The cohort structure may stretch over two semesters or may meet extra hours each week. Credit hours assigned may depend on required hours and/or required content. Students may receive one grade for embedded coursework or separate grades for separate courses. Separate courses may be taught by two different instructors.

Due to the complexity of various corequisite models, it is recommended that each campus employ a Corequisite Program Coordinator, assigned the responsibilities of scheduling paired courses, recruiting course instructors, monitoring the success of the chosen model, and communicating with all other appropriate parties on campus.

Professional Development and Placement

Professional Development

Teaching courses with underprepared students "differs substantially from teaching more advanced college-level math courses," requiring faculty to be highly qualified, not only in the discipline of mathematics, but also in content pedagogy and andragogy (Bonham & Boylan, 2011, p. 16). For this reason, specialized professional development should be provided for faculty offering just-in-time social, emotional and intellectual support. The relationship between cognitive and non-cognitive or affective factors can impact student success and should not be disregarded. Low self-esteem, lack of confidence, attitude, and mathematics anxiety are barriers to student success and must be acknowledged and addressed by faculty instructing a support course. Adjunct faculty should also be included in the process of developing program practices, implementation, and professional development opportunities.

Placement

There are many important considerations in placing students into the appropriate college math course. The idea that a single test with a single cut score can be used to accurately place a student into a college-level versus a developmental level mathematics course is being challenged by many sources. Multiple measures offer a more accurate way for students to demonstrate their potential to succeed in college-level courses. For example, the Community College Research Center (CCRC) found that a student's high school grade point average (GPA) often is a better predictor of college-level class performance than relying solely on their standardized assessment scores.

Jobs for the Future, The Charles A. Dana Center at The University of Texas at Austin, and Achieving the Dream released a Call to Action report for states and colleges to revise their math placement policies, processes, and supports. One goal was to ensure that students interested in a STEM field enter and complete the math courses they need. The report offers six recommendations designed to change how educators approach student math placements for math pathways.

1. Begin the placement support process early to ensure entering students are ready for college-level math.

2. Use multiple factors to determine whether students should be placed into developmental courses and which developmental or gateway courses are most appropriate.

3. Require test makers to align placement tests with differentiated math pathways and improve their predictive value.

> 4. Strengthen the role of student supports—especially advising—in the placement process.
>
> 5. Prioritize student academic and career goals in the placement process. In particular, keep STEM-aspiring students on STEM pathways.
>
> 6. Create a bridging mechanism from non-algebra pathways into algebra pathways (Couturier, L. K., & Cullinane, J., 2015, pp. 8-11).

Complete College America, a national nonprofit, recommends the following with regard to mathematics placement in its report, “The Game Changers: Are states implementing the best reforms to get more college graduates?”

> Use a placement range, not a single cut score. High-stakes placement exams have been proven to be poor predictors of college readiness, unnecessarily sending thousands of students into remediation each year.
>
> Instead, use a placement range to start most underprepared students in college-level courses with corequisite academic support, within which 75 percent or more of those students can succeed. In essence, establish two cut scores: one that provides direct entry into standard college courses and another that signals very low level of readiness for college work, even with corequisite assistance.
>
> Multiple measures should be used to provide a complete understanding of student ability. Students should also be given the opportunity to prepare for placement exams with practice tests and prep sessions (Complete College America, 2013, p. 10).

Missouri institutions are tasked to use multiple measures rather than a single placement score or exam. Examples of exams and metrics that are used across the state include ACT scores, high school GPA, Accuplacer, ALEKS PPL (Placement, Preparation, and Learning), declared meta-majors, whether a student took three or four years of high school mathematics and the GAIN test.

Complete College America recommends that students be given the opportunity to review for any exam that will result in placement into mathematics courses. It is also advisable that institutions develop a long-term assessment plan to determine whether or not cut scores need to be adjusted.

Corequisite Instruction

Curriculum and Instructional Strategies

The purpose of the corequisite course is to provide students with just-in-time support in order to assist them in meeting the Student Learning Outcomes (SLO) established by the college-level course. Corequisite course instructors should provide opportunities for underprepared students to preview upcoming college-level content, practice particular skills, identify misconceptions, and review concepts in greater depth. Instructional schedules for the corequisite course should parallel that of the paired college-level course.

The college-level SLO should drive the instructional schedule and practices of the corequisite course. In the companion course model both courses should be closely aligned. It is imperative that instructors of both the college-level course and the corequisite course participate in frequent communication and collaboration. Students and instructors of both courses will benefit from sharing information regarding course objectives, materials, schedules, and expectations along with student participation, attendance, and performance. It may be helpful for the instructor of the general education course to provide the corequisite instructor "teaching assistant" permissions in the learning management system (LMS) so that the corequisite instructor has access to resources, materials and schedules just as the students do.

Underprepared students benefit from a variety of instructional strategies. Successful corequisite support programs should utilize multiple teaching and active-learning strategies. Suggested strategies include, but are not limited to:

- Cooperative and collaborative learning structures;
- Integration of math study skills and learning strategies such as time management, organizational skills, test preparation, note taking methods and finding resources;
- Problem solving and modeling activities;
- Integration of technology;
- Project-based instruction;
- Tasks focused on building mathematical language;
- Course discussions.

A Specific Example

MATH 101 Precalculus Algebra paired with MATH L1 Support Course

Having communicated the instructional schedule with the instructor of MATH L1 prior to the lecture, the instructor of MATH 101 plans to deliver a lecture on Zeros of Polynomial Functions on Wednesday. On Tuesday, prior to the lecture, the instructor of MATH L1 might use a Kahoot or other interactive activity to pre-assess and to review terms and skills related to factoring, x-intercepts, degree of polynomials, synthetic division, etc. The instructor of MATH L1 could follow this activity by providing practice problems involving synthetic division and factoring in order to build student skills. Both activities will allow the MATH L1 instructor opportunities to identify student strengths and weaknesses that can be addressed prior to the lecture. On

Thursday, following the MATH 101 lecture, students in MATH L1 might participate in a small group Card Sort activity requiring students to match graphs with corresponding functions to reinforce the concept of polynomial zeros. Students could also participate in an interactive online poll using True/False statements as a prompt for whole class discussion.

In this example, students have received a typical college-level lecture in MATH 101 while also participating in a variety of learning activities in MATH L1. Students benefit from increased exposure to content through a variety of strategies along with additional opportunities to ask questions and practice skills.

Evaluation Systems

There are various methods of implementing the corequisites, each unique to the needs of individual institutions. The primary purpose of the corequisite course is to assist students in developing knowledge and skills necessary for success in the general education course. Therefore, the student learning outcomes of the corequisite course should mirror those of the general education course.

Due to the design of corequisite courses, formative assessments should be used to guide teaching and learning strategies, while summative assessment should be used in the general education course to measure student achievement on the student learning outcomes (SLOs). Furthermore, it is recommended that the summative assessment scores, earned in the general education course, be included in the course grading system of the corequisite rather than testing students twice over the same content. Attendance in the corequisite course should be mandatory and should be included in the course grading policy. Instructors may want to include learning tasks, practice work, projects, portfolios, technology enhanced assignments, discussion posts, and other student work in the evaluation system. A traditional A, B, C, D, F grading system can be used for the corequisite course or it could be graded as pass/fail.

Resources for Instructors

1. The Charles A. Dana Center at the University of Texas at Austin
The Dana Center seeks to ensure that ALL students in higher education will be:

- **Prepared** to use mathematical and quantitative reasoning skills in their careers and personal lives
- **Enabled** to make timely progress towards completion of a certificate or degree
- **Empowered** as mathematical learners.

The Dana Center Math Pathways Resource site has information on corequisite models preparing students for the appropriate math pathway, including the following document:
Co-requisite Remediation: Narrowing the Gap between Instruction and Supports.
https://dcmathpathways.org/

2. The National Association of Developmental Educators (NADE)

NADE seeks to improve the theory and practice of developmental education at all levels of the educational spectrum, the professional capabilities of developmental educators, and the design of programs to prepare developmental educators.
https://thenade.org/

3. The Community College Research Center (CCRC), housed at Teachers College, Columbia University

CCRC conducts research on the issues affecting community colleges and works with colleges and states to improve student success and institutional performance. CCRC produces working papers, research reports, practitioner packets, and other resources aimed at promoting student success.
https://ccrc.tc.columbia.edu/

4. Complete College America (CCA)

Established in 2009, Complete College America is a national nonprofit with a single mission: Leveraging our Alliance to eliminate achievement gaps by providing equity of opportunity for all students to complete college degrees and credentials of purpose and value.
https://completecollege.org

Open Educational Resources (OER) for the Classroom

OERs offer low cost or free course materials so that all students have access.

1. OpenStax, Rice University

OpenStax is a nonprofit based at Rice University, and its mission is to improve student access to education by providing openly licensed free digital college textbooks. The Openstax library currently includes eleven math textbooks, including Introductory Statistics, Pre-calculus and developmental level mathematics titles.
https://openstax.org/

OpenStax Partners:
OpenStax Partners create optional low-cost technology products that are integrated with the OpenStax books.
https://openstax.org/partners

a. Knewton Alta, $44 per course

Knewton offers a personal learning experience to every student by dynamically adapting to each student's unique needs. Knewton provides students with tailored recommendations for exactly what to study, and has corequisite support topics integrated into its Pre-calculus, Introductory Statistics, and Quantitative Reasoning courses.
https://www.knewtonalta.com/

2. Lumen Learning

Lumen Learning creates digital course materials that replace expensive textbooks in high-enrollment college courses and save students millions every term.

a. Lumen OHM, $25 per course

Lumen OHM instructors can configure and customize learning activities by replicating or adjusting course materials. Ten editable math texts are available in Lumen OHM including Precalculus, Math for Liberal Arts, Introductory Statistics as well as developmental level math courses.
https://lumenlearning.com/

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Recommended Published Works

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The Missouri Corequisite at Scale Taskforce Recommended Competencies

The Missouri Math Pathways Taskforce determined Student Learning Outcomes as the minimum requirements of a credit-bearing, entry-level, college course in mathematical reasoning. Driving forces for the development of Student Learning Outcomes included the development of alternative, entry-level courses specific to academic majors and increased course transferability statewide.

The Missouri Corequisite at Scale Taskforce offers the Recommended Competencies as *possible* topics of study for a mathematical reasoning, statistical reasoning or precalculus algebra corequisite-course. These topics include just-in-time learning of foundational skills and review of credit-bearing course content.

Mathematical Reasoning Recommended Competencies

Possible Corequisite Topics	Student Learning Outcomes
The Corequisite at Scale Taskforce offers the following *possible* topics of study for a mathematical reasoning, corequisite course. These topics include just-in-time learning of foundational skills and review of credit-bearing, course content.	The Missouri Math Pathways Taskforce has determined the following Student Learning Outcomes as the minimum requirements of a credit-bearing, entry-level, college course in mathematical reasoning.

I. Proportional Reasoning

Students will draw conclusions or make decisions in quantitative-based situations using proportional reasoning. Specifically, students will be able to:

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Simplify and evaluate algebraic expressions • Use order of operations • Use estimation • Simplify fractions • Write fractions/percentage in decimal forms • Find a percentage increase/decrease • Perform operations with fractions and decimals • Solve linear equations • Solve proportion equations • Solve linear inequalities	• Use ratios, proportions, rates and percentages to explain, draw conclusions, or make decisions. • Use units and unit conversions to explain, draw conclusions, or make decisions.

II. Statistical Reasoning **Students will read, interpret, analyze and synthesize quantitative data (e.g., graphs, tables, statistics, survey data, etc.) and make reasoned estimates and inferences. Specifically, students will be able to:**	
Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Find the slope of a line • Determine an equation of a line • Find the intercepts of a line and interpret their meaning • Substitute values and evaluate an expression • Use exponential notation and properties • Use radicals • Plot points in the Cartesian Coordinate System • Graph linear equations by plotting points • Use subscript and summation notation • Shade a described area	▪ Collect and organize data in graphs and tables. ▪ Use descriptive statistics to interpret and analyze quantitative data. ▪ Use probability to interpret and analyze quantitative data. ▪ Communicate statistical findings effectively.

III. Mathematical Modeling **Students will create, apply and use mathematical models to solve problems. Specifically, students will be able to:**	
Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Plot points in Cartesian Coordinate System • Solve a system of equations graphically • Translate phrases into mathematical expressions • Translate applications into equations • Solve linear inequalities in two variables • Graph exponential functions • Find the slope of a line • Use exponential notation and properties • Use logarithms and properties • Use order of operations • Simplify radicals • Evaluate complex expressions using technology	▪ Describe and contrast linear rate and non-linear rate through verbalization and writing. ▪ Create linear and exponential functions from quantitative data and explain the results. ▪ Interpret and analyze linear and exponential functions that model data.

Statistical Reasoning Recommended Competencies

Recommended Competencies	Student Learning Outcomes
The Corequisite at Scale Taskforce offers the following *possible* topics of study for a statistical reasoning, corequisite course. These topics include just-in-time learning of foundational skills and review of credit-bearing, course content. Instruction on the efficient use of technology and study skills are also advised.	The Missouri Math Pathways Taskforce has determined the following Student Learning Outcomes as the minimum requirements of a credit-bearing, entry-level, college course in statistical reasoning.

I. Data Exploration

Students will analyze data using graphical and numerical methods to study patterns and departures from patterns, using appropriate technology as needed. Specifically, students will be able to:

Construct and interpret graphical displays of distributions of univariate data.

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Plot points and intervals on a number line • Perform signed number arithmetic • Read to understand information from tables and graphs	▪ Create and interpret dotplots, boxplots, stem and leaf plots and histograms. ▪ Analyze center, shape and spread, as well as clusters, gaps, outliers and other unusual features.

Summarize distributions of univariate data and compare multiple distributions.

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Use summation notation • Plot an ordered pair (x, y) in a rectangular coordinate system • Round decimal values • Understand powers and square roots of numbers • Understand order of operations	▪ Compute measures of center (median, mean), measures of spread (range, interquartile range, standard deviation) and measures of position (quartiles, other percentiles and standardized scores). ▪ Compare groups using back-to-back stem and leaf plots, parallel boxplots and dotplots.

Explore bivariate data.

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Find the slope of line segment connecting two points, the equation of a line, and graph the equation of a line • Find the vertical distance between a point and a line • Round decimal values	▪ Analyze scatterplots for patterns, linearity, and outliers. ▪ Calculate and interpret the correlation coefficient.

I. Data Exploration (continued)	
Students will analyze data using graphical and numerical methods to study patterns and departures from patterns, using appropriate technology as needed. Specifically, students will be able to:	
Explore categorical data.	
Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Read to understand information from a table or a graph	▪ Create and interpret frequency tables and bar charts. ▪ Compare distributions of categorical data.

II. Statistical Design	
Students will critically evaluate a data-collection plan to answer a given research question. Specifically, students will be able to:	
Identify characteristics of good study designs. Understand what conclusions are appropriate for a given design and whether conclusions can be generalized to a larger population.	
Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Read carefully through a problem • Know and understand key terms • Read carefully to identify important information in a word problem	▪ Identify the population of interest. ▪ Determine whether an observational or experimental study is appropriate and feasible. ▪ Explain the difference between and importance of random selection and random assignment in study design.
Know the elements of planning and conducting an observational study.	
Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Distinguish between a sample and a population • Differentiate between key terms	▪ Verify basic elements of statistically valid sample survey. ▪ Determine when a census or a sample survey is appropriate. ▪ Identify potential sources of bias in sampling and surveys.
Know the elements of planning and conducting an experimental study.	
Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Recognize and differentiate between key terms • Identify dependent and independent variables	▪ Verify basic elements of statistically valid experimental design. ▪ Explain the purpose of including a control group and blinding in an experiment. ▪ Identify potential sources of confounding in an experiment.

III. Probability and Simulation Students will use probability concepts and simulation. Specifically, students will be able to:	
Determine and interpret probabilities.	
Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Convert among fractions, decimals, and percents • Operate with fractions	▪ Interpret a probability as a long-run relative frequency of occurrence. ▪ Calculate the probability of a specified event in a chance experiment with equally likely outcomes.
Use probability distributions to describe the behavior of discrete and continuous random variables.	
Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Decide upon appropriate units of measurement in collection data • Perform signed number arithmetic • Plot numbers on a real number line, find a mean value and a range • Represent an inequality as an interval on the number line • Shade an area under the normal distribution	▪ Distinguish between discrete random variables and continuous random variables. ▪ Compute and interpret the mean and standard deviation of the probability distribution of a discrete random variable. ▪ Demonstrate an understanding of the mean, standard deviation and shape of continuous probability distributions (uniform, normal and skewed).
Understand distributions.	
Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Recognize and differentiate between key terms	▪ Distinguish between the distribution of a sample and a sampling distribution. ▪ Describe the sampling distributions of a sample mean and sample proportion in terms of center, shape and spread. ▪ Explain how these relate to sample size. ▪ Identify when the use of the normal distribution is appropriate.

IV. Statistical Inference

Students will use statistical models to draw conclusions from data. Specifically, students will be able to:

Estimate population parameters including confidence intervals when appropriate.

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Understand order of operations • Realize properties of inequalities	▪ Verify that the appropriate conditions have been met. ▪ Construct one-sample confidence intervals for means and for proportions. ▪ Construct two-sample confidence intervals for means. ▪ Interpret confidence intervals in context and explain the meaning of the confidence level associated with a confidence interval estimate.

Conduct tests of significance when appropriate.

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Understand order of operations • Represent an inequality as an interval on the number line • Interpret probability • Use function notation	▪ Verify that the appropriate conditions have been met. ▪ Carry out one-sample hypothesis tests for means and proportions. ▪ Carry out two-sample hypothesis tests for means. ▪ Interpret the meaning of rejection of the null hypothesis and of failure to reject the null hypothesis, in context. ▪ Demonstrate an understanding of the use of a p-value to reach a conclusion and of the difference between practical significance and statistical significance.

V. Regression Modeling

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Find the slope of line segment connecting two points, the equation of a line, and graph the equation of a line • Understand slope as a ratio of change	▪ Determine the equation of the least-squares regression line and interpret its slope and intercept in context.

Precalculus Algebra Recommended Competencies

Recommended Competencies	Student Learning Outcomes
The Corequisite at Scale Taskforce offers the following *possible* topics of study for a Precalculus Algebra, corequisite course. These topics include just-in-time learning of foundational skills and review of credit-bearing, course content.	The Missouri Math Pathways Taskforce has determined the following Student Learning Outcomes as the minimum requirements of a credit-bearing, entry-level, college course in precalculus algebra reasoning.

I. Foundations of Functions

Students will use multiple representations of different function types to investigate quantities and describe relationships between quantities. Specifically, students will be able to:

Use multiple representations of functions to interpret and describe how two quantities change together.

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Use interval notation • Interpret radical and rational expressions • Evaluate functions • Apply the order of operations • Recognize relationship between inputs and outputs of functions • Sketch graphs of common functions • Interpret inequality symbols	▪ Identify constraints on quantities and domains. ▪ Distinguish dependent and independent variables. ▪ Identify domains and ranges. ▪ Effectively communicate using function notation.

Measure, compute, describe and interpret rates of change of quantities embedded in multiple representations.

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Calculate and interpret slope • Explain a rate of change in terms of slope including appropriate units	▪ Identify constant rates of change. ▪ Determine average rates of change. ▪ Be able to estimate instantaneous rates of change.

Use appropriate tools and representations to investigate the patterns and relationships present in multiple function types.

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Graph basic functions • Solve algebraic equations • Identify domains and ranges • Use calculators or computer software in accordance to requirements in Pathways course	▪ Work effectively with the following functions: linear, quadratic, exponential, logarithmic, rational, piecewise and absolute value.

II. Analysis of Functions

Students will describe characteristics of different function types and convert between different representations and algebraic forms to analyze and solve meaningful problems. Specifically, students will be able to:

Create, use and interpret linear equations and convert between forms as appropriate.

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Read information from graphs, figures, tables, etc. • Perform operations with fractions	▪ Identify important values (i.e. slope and intercepts) from multiple representations. ▪ Determine equations of lines given one point and the slope, two points or statements about proportional relationships.

Create, use and interpret exponential and logarithmic equations and convert between forms as appropriate.

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Use properties of logarithms • Use rules of exponents • Understand inverse relationships between exponential and logarithmic functions	▪ Explain exponential growth as constant percentage rate of change. ▪ Interpret half-life and doubling time to create decay and growth models. ▪ Recognize similarities and differences between linear and exponential functions. ▪ Recognize the role of *e* as a natural base ▪ Describe long-term behavior of exponential models. ▪ Apply the inverse relationship between exponential and logarithmic functions.

Create, use and interpret polynomial, power and rational functions.

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Perform operations on polynomials • Create graphs of basic functions • Solve algebraic equations • Identify where a function is increasing, decreasing, or constant • Use the quadratic formula • Find domain of rational functions • Use function notation (i.e. evaluate $f(-x)$)	▪ Recognize how power functions are different from exponential functions. ▪ Determine whether a graph has symmetry and whether a function is even or odd. ▪ Determine end behavior, maximum, minimum and turning points of a graph. ▪ Find roots of a function and correctly graph the function. ▪ Graph rational functions and find vertical, horizontal and oblique asymptotes.

II. Analysis of Functions (continued)	
Students will describe characteristics of different function types and convert between different representations and algebraic forms to analyze and solve meaningful problems. Specifically, students will be able to:	
Construct, use and describe transformations, operations, compositions and inverses of functions.	
Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• State the domains and ranges of functions • Perform operations on functions • Perform integer operations • Define the term "function" • Sketch the graphs of basic functions • Solve for the indicated variable	▪ Describe how the graph of a function can be the result of vertical and horizontal shifts, stretches, compressions, and reflections of the graph of a basic function. ▪ Perform arithmetic operations with functions and describe the domain. ▪ Create new functions by composing basic functions and describe the domain. ▪ Decompose a composite function into basic functions. ▪ Determine if a function is one-to-one, and if so, find the inverse and describe its domain and range.

III. Algebraic Reasoning

Students will identify and apply algebraic reasoning to write equivalent expressions, solve equations and interpret inequalities. Specifically, students will be able to:

Use algebraic techniques to simplify expressions and locate roots.

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Factor polynomials • Solve algebraic equations • Simplify radicals • Solve compound inequalities • Graph inequalities in one variable • Perform integer operations • Define the imaginary unit	▪ Solve quadratic equations by factoring, the square root property, completing the square, and the quadratic formula. ▪ Solve quadratic, absolute value, polynomial and rational inequalities. ▪ Perform operations with complex numbers. ▪ Determine complex roots of polynomials.

Use algebraic reasoning to simplify a variety of expressions and find roots of equations involving multiple function types.

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Use rules of exponents • Solve algebraic equations	▪ Apply properties of exponents and logarithms. ▪ Solve polynomial, radical, rational, exponential, and logarithmic equations.

Use rational exponents to express and simplify a variety of expressions and solve equations.

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Use rules of exponents • Identify and factor out the greatest common factor	▪ Factor out common rational powers. ▪ Simplify fractional expressions involving rational exponents.

Solve and apply systems of equations and inequalities.

Possible Corequisite Topics	*Pathways Initiative Student Learning Outcomes*
• Translate written statements into algebraic equations • Perform operations on real numbers • Solve equations	▪ Set up and solve systems of equations. ▪ Perform matrix operations. ▪ Use matrices to solve systems of linear equations. ▪ Graph systems of inequalities.